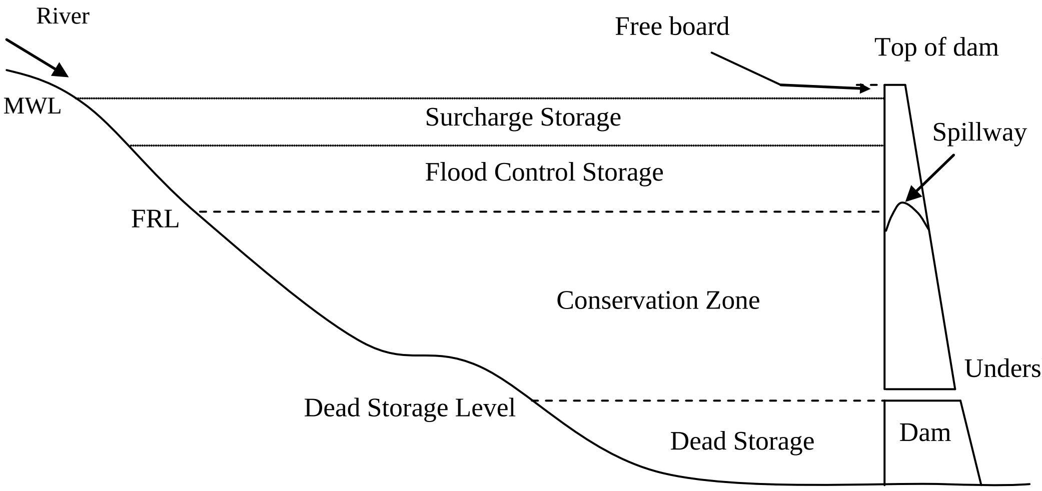
DETERMINATION OF RESERVOIR STORAGE CAPACITY

Sharad K Jain
Scientist G
National Institute of Hydrology
Roorkee 247667, India
s_k_jain@yahoo.com

1.1 Determination of Reservoir Storage Capacity

The storage capacity of a reservoir is conceptually divided into a number of zones based on the useful purposes that a reservoir is required to serve. Fig. 1.1 gives a schematic of various storage zones of a reservoir. Dead storage zone is the bottom-most zone of a reservoir. Major storage space is occupied by the conservation zone. If the reservoir is operated to control floods then the flood control storage is provided above the conservation zone flowed by the surcharge storage. The procedure for fixing size of various storage zones is described in the following.



FRL: Full reservoir level
MWL: Maximum water level

Fig. 1.1 Schematic diagram of a reservoir showing the various storage zones.

1.2 Dead Storage Capacity

Dead storage is provided in a reservoir chiefly to serve two purposes:

a) The river, during its course to the reservoir, picks up sizeable amount of sediment and carries it along either as suspended load or bed load. Upon entering a reservoir, the velocity of flow becomes virtually zero and hence its carrying capacity is lost. So the sediment settles down and it keeps on accumulating as the time passes on. On account of this accumulation, the effective storage capacity of the reservoir and hence its reliability goes on reducing with time. Dead storage is the zone where these sediments mainly settle (they settle in other zones also).

b) Many times the water released from the reservoir is passed through the turbines of power plants located downstream of the dam to generate hydroelectric power. For efficient working of turbines, it is necessary that head variation must be within a specified range and a minimum head must always be available. This minimum head corresponds to the top of the dead storage zone.

These two considerations necessitate the provision of dead storage in the reservoir. To compute the amount of sediment inflow expected in the reservoir, average sediment yield of the catchment is determined. This data is then used to compute sediment deposition expected during the economic life of the project. The storage actually provided in the reservoir is covered by the greater of the two factors discussed above. Bottom outlets are provided above the sediment deposition level.

1.3 Storage Requirement for Conservation Purposes

A number of techniques are available for computing storage capacity for conservation purposes like irrigation, municipal and industrial water supply, hydropower generation etc. Depending upon the type of data and the computational technique used, the popularly used reservoir capacity computation procedures are classified into following categories:

a) Critical Period Techniques

The techniques based upon critical period concepts are the earliest techniques of storage-yield analysis. The critical period is defined as the period in which an initially full reservoir, passing through various states (without spilling), empties. One such method, known as the *Mass Curve Method* was the first rational method proposed to compute the required storage capacity of a reservoir. The other popular method in this group is the *Sequent Peak Method*.

b) Simulation/Optimization Techniques

With the advent of computer, the techniques, which beneficially use its computational capabilities are increasingly being used. Among the optimization techniques, those based on Linear Programming (LP) have been found to be particularly suitable for reservoir design and determination of optimal cropping pattern, etc. The simulation approach is a very powerful technique that can be used as a stand-alone method for reservoir design or can be used to further modify and test the results of other methods.

c) Probability Matrix Methods

These methods use statistical laws to analytically solve the storage-yield analysis problems. Well known methods in this class are the Moran's method and the Gould's method. These methods are not used in practice.

1.3.1 The Mass Curve Method

This method is also known as the Rippl mass curve method after the developer of this method. This is a simple method which is commonly used to estimate the required storage capacity of a reservoir in project planning stage. The method uses the most critical period of recorded flow to compute storage. The critical period is defined as the duration in which initially full reservoir depletes and passing through various states empties without spilling. In the methods based on critical period concept, a sequence of streamflows containing a critical period is routed through an initially full reservoir in presence of specified demands. The reservoir capacity is obtained by finding the maximum difference between cumulative inflows and cumulative demand curves. Let x(t) be the inflows to a reservoir in volumetric units. We define a function X(t):

$$X(t) = \Sigma_t \, x(t) \, dt \tag{1.1}$$

then the graph of X(t) versus time is known as the mass curve. The mass curve technique, proposed by Rippl in 1883 to determine storage capacity of a reservoir, is a graphical technique.

To determine minimum required storage, the mass curve of inflow and the mass curve of demand are accumulated separately. For a constant draft, the mass curve of demand will be a straight line having a slope equal to the demand rate. Now, at each high point on the mass inflow curve, a line is drawn parallel to the mass curve of demand and is extended until it meets the mass curve of inflows. For illustration, the mass curve of inflows for a reservoir is plotted in Fig. 1.2 by the thick line. The line AB is the mass curve of demands. Two lines, namely line CD and EF, are drawn such that they are parallel to line AB and are tangent to the mass curve of inflows at points C and E. The maximum vertical distance between the mass curve of inflows and line CD and EF is noted. Similar procedure is repeated for all peak points on the mass curve. The maximum of these vertical distances between the mass curve of inflows and the mass curve of demands is the required storage.

Fig. 1.2 Mass curve method for determination of storage

Although the mass curve technique is very simple and straight forward, it has a few shortcomings. This method is suitable when the draft is constant. It is not possible to consider evaporation losses. One drawback is the implicit assumption that the storage which would have been adequate in past will also be adequate in future. Although this is not clearly true, the error caused is not really serious particularly if sufficiently long flow series has been considered.

However, this problem will arise in any other method since true future is not known. Some methods try to address this problem by explicitly considering the stochasticity of the inflows. One more drawback of the mass curve is that no economic analysis can be done in this technique. The storage size cannot be related to the economic life of the project and usually estimate of the storage increase with the increase in the length of record used. Further, storage size cannot be computed for a particular level of reliability.

Mass curve method has a number of strengths. The main of these is that the method is simple and very intuitive. Perhaps these are the reasons of its popularity and wide use.

1.3.2 Sequent Peak Algorithm

Sequent Peak Algorithm overcomes some shortcomings associated with the mass curve method. This method is particularly suited for the analysis of large data with the help of a computer. It was proposed as a method which circumvents the need to choose the correct starting storage which is required in the mass curve procedure. The computations are quite simple and can be carried out as follows.

Let I_t be the inflow to the reservoir in the period t, R_t be the release from the reservoir, and S_t the storage at the beginning of the period t. The reservoir is assumed to be empty in the beginning. The mass curve of cumulative net flow volume (Inflow - Outflow) against chronological time is used. This curve will have peaks (local maximums) and troughs (local minimums). For any peak P_i the next following peak of magnitude greater than P_i is called a sequent peak. The computations are performed for twice the length of the inflow record by assuming that the inflows repeat after the end of first cycle. This assumption is made to take care of the case when the critical period falls at the end of the record.

The variable S_t is calculated by the following equation:

$$S_t = \begin{vmatrix} S_{t-1} + R_t - I_t & \text{if positive} \\ 0 & \text{if negative or zero} \end{matrix} \qquad (1.2)$$

The required storage capacity is equal to the maximum of S_t values.

Example 1.1: A reservoir is to be constructed at a location where monthly flow data are available for 28 months. It is required to release 35 MCM of water from the reservoir every month. Find the minimum size of the required reservoir by the Sequent Peak Algorithm.

Solution: The computations are illustrated in the following table where the inflows are as given in column 3.

All values are in million cubic m (MCM).

Period (t)	Storage S_{t-1}	Inflow I_t	Release R_t	Storage S_t

1	0.0	17.1	35	17.9
2	17.9	47.2	35	5.7
3	5.7	76.7	35	0.0
4	0.0	2.6	35	32.4
5	32.4	0.7	35	66.7
6	66.7	0.0	35	101.7
7	101.7	0.0	35	136.7
8	136.7	0.0	35	171.7
9	171.7	0.6	35	206.1
10	206.1	0.1	35	241.0
11	241.0	0.7	35	275.3
12	275.3	0.0	35	310.3
13	310.3	6.2	35	339.1
14	339.1	10.1	35	364.0
15	364.0	40.7	35	358.3
16	358.3	0.6	35	392.7
17	392.7	0.0	35	427.7
18	427.7	0.0	35	462.7
19	462.7	0.3	35	497.4
20	497.4	0.2	35	532.2
21	532.2	0.4	35	566.8
22	566.8	0.0	35	601.8
23	601.8	0.3	35	636.5
24	636.5	0.0	35	671.5
25	671.5	8.7	35	697.8
26	697.8	184.9	35	547.9
27	547.9	527.2	35	55.7
28	55.7	48.1	35	42.6
29	42.6	17.1	35	60.5
30	60.5	47.2	35	48.3
31	48.3	76.7	35	6.6
32	6.6	2.6	35	39.0
33	39.0	0.7	35	73.3
34	73.3	0.0	35	108.3
35	108.3	0.0	35	143.3
36	143.3	0.0	35	178.3
37	178.3	0.6	35	212.7
38	212.7	0.1	35	247.6
39	247.6	0.7	35	281.9
40	281.9	0.0	35	316.9
41	316.9	6.2	35	345.7
42	345.7	10.1	35	370.6
43	370.6	40.7	35	364.9
44	364.9	0.6	35	399.3
45	399.3	0.0	35	434.3
46	434.3	0.0	35	469.3
47	469.3	0.3	35	504.0
48	504.0	0.2	35	538.8
49	538.8	0.4	35	573.4
50	573.4	0.0	35	608.4
51	608.4	0.3	35	643.1
52	643.1	0.0	35	678.1
53	678.1	8.7	35	**704.4**
54	704.4	184.9	35	554.5
55	554.5	527.2	35	62.3
56	62.3	48.1	35	49.2

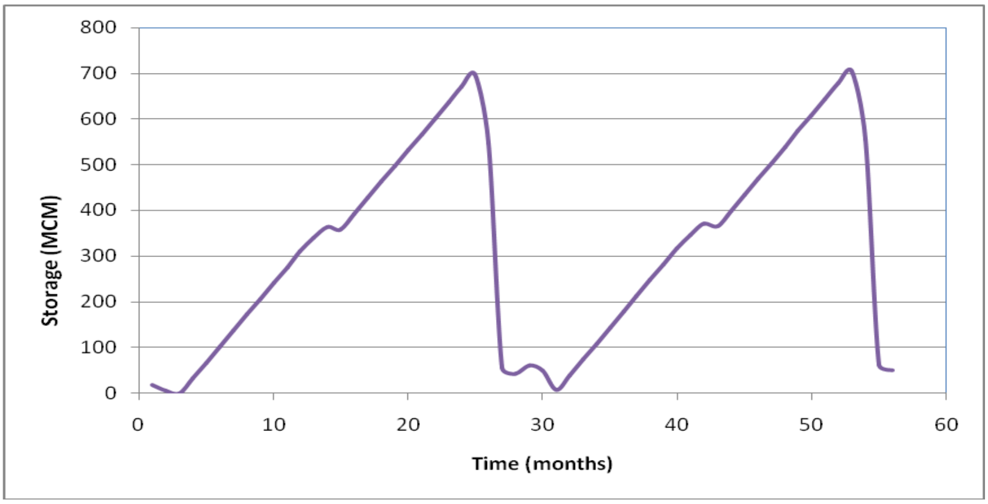


Fig. 1.3 Sequent peak algorithm

The required storage is given by the maximum of the values in the last column which is 704.4 MCM. Here the calculations have been repeated for the second cycle of the inflows. Fig. 9.3 shows the graphical presentation of the method and the concept of sequent peak.

In the sequent peak algorithm, it is very easy to consider the variable release from the reservoir. The reliability of the reservoir can be obtained indirectly. Since the reservoir would be able to meet the worst drought from the record, the implied probability of failure would be 1/(N+1).

Sequent peak algorithm is very fast and easy to program. A single historical record is used to compute the storage and hence the method is limited in that sense. It is also not possible to exactly consider the losses, these can be approximately included in the releases.

1.3.3 Storage-Yield Analysis

Storage yield analysis is used to determine the volume of reservoir storage required to augment river flow in order to provide a specified water demand with a stated reliability. It is also used to reassess the water demand which can be satisfied by existing reservoirs. Storage volume depends upon the volume of demand D, specified reliability R and the hydrograph of the catchment supplying the reservoir. Reliability R is an index in the range 0 - 1 which indicates how satisfactorily the reservoir performs. If storage required is to be calculated then yield is known otherwise the storage capacity is known. The Fibonacci search technique is used for the computation of dependent variable, reservoir capacity or annual yield, till desired reliability is achieved with permissible tolerances, supplied by the user.

When a sufficient long record of monthly or annual flows is available, then analysis of that series using suitable methods can provide the required storage capacity estimates once the levels of demand and reliability are specified. The following steps are followed:

a) At the beginning of iteration, the upper bound of the variable is kept equal to the average inflow volume in a year. The lower bound of storage is taken as dead storage S_{min}, whereas for annual yield lower bound is taken as zero.
b) Reservoir is initially assumed to be full.
c) Continuity eqn. is applied for each time unit

$$S_{t+1} = S_t + I_t - E_t - L_t \qquad (1.3)$$

d) The resulting Storage value series can be plotted versus time to show the behavior of the reservoir for the chosen trial capacity.
e) From above results, reliability is calculated.
f) If these values are too small, a large capacity is chosen and steps 1 to 5 are repeated.
g) If reliability values are large, and a smaller value is acceptable, then a smaller capacity is chosen and steps 1 to 5 are repeated.
h) This trial and error is performed till desired value of reliability is achieved.

With the desired accuracy, specified lower bound and calculated upper bound, one dimensional search is carried out to reach the optimum value of variable. The reliability achieved is computed after complete reservoir operation computations, based on mass balance equation. The evaporation loss E_i is function of both S_i and S_{i+1}. Hence an iterative method is applied using elevation-area-capacity table till absolute difference between two successive relative evaporation losses are less than a value supplied by the user. At each time interval, attempt is made to satisfy the demand to the extent possible. If the available water in reservoir is less than S_{min}, no release is made and the storage is depleted by evaporation only and the reservoir is assumed to have failed during that particular month. If during any period, $S_i + I_i$ □ C, the extra water over the storage capacity after meeting the demand is spilled. If there is not enough water in the reservoir to meet the demand any period, the demand is met to the extent possible and the month is treated as failure month.

The reliability achieved (REL) is computed by

$$REL = 1.0 - FAIL/n \qquad (1.4)$$

where FAIL = number of failures (number of periods when $R_i < D_i$). The objective function used in Fibonacci search is

$$OF = (REL – RELI) \qquad (1.5)$$

where RELI is the reliability desired.

The detail of Fibonacci search method, which is a unidirectional search method for nonlinear optimization problems, can be found in texts such as Rao (1992). The choice of this method over other univariate nonlinear programming techniques is somewhat subjective.

1.4 Optimization Techniques

The advent of computer and the development of optimization techniques has led to the use of both of these to storage-yield analysis. Among the various available optimization techniques, Linear Programming (LP) and Dynamic Programming (DP) are two techniques which have been used extensively. Here, only a LP based formulation is being discussed. The problem formulation is essentially same in case of DP.

Let us consider a situation in which a reservoir is to be constructed at a particular site. Monthly inflow data for past n months is available. The projected demand of water during a critical year is known along with its distributions among each month. The losses from the reservoir are neglected for the time being. The problem is to find out the minimum capacity of reservoir which will supply the required quantity of water without failure. Let X be the annual water demand from the reservoir and α_i, i = 1,2..... 12 be its fractions for different months. Hence the demand in a particular month will be α_i X. Let I_i be the inflow to the reservoir during the i^{th} month and R_i be the water actually released from the reservoir.

Representing by S_i, the storage content of the reservoir at the beginning of month i, the continuity equation is:

$$S_i + I_i - R_i = S_{i+1} \tag{1.6}$$

This equation has to be satisfied for each of the n months and hence we shall have n such equations which will be constraints in the formulation. The value of S_i is given as input.

It is also required that the amount of water actually released from the reservoir must be more than or equal to the amount demanded. This can be mathematically expressed as:

$$R_i \geq \alpha_i X \quad i = 1, 2......n \tag{1.7}$$

Since this condition also must hold for each month, there will be n such constraints. If the capacity of the required reservoir is C, then in any month, from physical point of view, the storage content of the reservoir must be equal to or less than this value. Hence

$$C \geq S_i \quad i = 1, 2......n \tag{1.8}$$

Moreover, the storage S_i, capacity C and release R_i can take only positive values. This completes the problem formulation. The problem is quite easy to solve particularly due to availability of standard package programs.

1.5 Simulation Method

Simulation is essentially a search procedure. It is one of the most widely used techniques to solve a large variety of problems associated with the design and operation of a water resources system. The reason is that this approach can be realistically and conveniently used to examine and evaluate the performance of a set of alternative options available. Further, serial correlation of inflows, seasonality etc. are easy to account. Also, it is easy to present the technique and its results to non-technical persons.

Assume that a site has been identified for the construction of a dam. The reservoir has to cater for irrigation for a nearby area and the target demand of water for different months is given. The elevation-area-capacity table for the site is available. A sufficiently long series of streamflows at the site is available. Further, it is required that the reliability of the reservoir should be least 75%. An efficient procedure of binary search can be used to determine the required storage capacity. In this method, first the upper and lower bounds on the capacity of the reservoir are determined. The lower bound can be taken to be zero or the dead storage and the upper bound can be determined from physical factors such as water availability etc. A trial value for the reservoir capacity is selected which is the mean of upper bound and lower bound.

Now, starting with a suitable value of initial storage content, the reservoir is operated using the streamflow data. The effect of this initial storage value will not be very significant if the inflow series for a sufficiently long period, say 30-40 years is being used. During any time period, the release is made equal to the demand if that much water is available in the storage. Otherwise whatever can be made available is released and the reservoir is said to have failed in that period. The evaporation losses can be easily considered if the information about the depth of evaporation is available. In this way, the reservoir is operated for the entire period of record. Now the reliability of the reservoir is computed. If this reliability is less than the desired value, it means that the capacity of the reservoir must be increased. In this case the present capacity is adopted as the lower bound for next iteration. The feasible region below this lower bound is discarded and the trial value for the next iteration is chosen midway the upper bound and new lower bound. If, however, the reliability comes out to be higher than the required limit, the size of the reservoir is bigger than what it should have been and hence the region between the current value and upper bound is discarded for further examination. The present capacity value becomes the new upper bound. Again the trial value for the next iteration is chosen as mean of new upper bound and old lower bound.

The computations are repeatedly performed in this manner and they are terminated when the required convergence is achieved. This method converges quite rapidly as the feasible region is halved every time. It may be seen that in this method, generation of hydroelectric power can also be easily considered.

One of the major criticism of this approach is that the analysis is based on historical flows which may not be representative of future conditions. Also, it is difficult to properly consider when the demands and releases are related to growth in time. Non-continuous records can also not be easily handled.

Although the method to be adopted for a particular problem will depend on the available data, the method of simulation has been rated as the best method for storage-yield analysis. Though simulation does not add any new information but it helps to extract the maximum amount of information from the available data. Undoubtedly, a sufficiently long and reliable inflow data series is the prime requirement for storage-yield analysis.

1.7 Summary

This lecture covers the methods that are used to determine storage capacity of a reservoir for meeting conservation demands. Discussion covers the conventional methods as well as those based on the use of optimization and simulation techniques.

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